Economic application of double integrals

The production function for an industry is given by $P(x, y) = 100 \cdot x^{0.6} \cdot y^{0.4}$, where x represents units of labor and y units of capital. Estimate the average level of production if x varies between 100 and 200 units and y varies between 300 and 350.

Solution

The formula for calculating the average value of a function is:

$$\frac{\int\int f(x,y)dydx}{A}$$

Where A is the area of the region enclosed by the limit values of x and y. First we compute the integrals:

$$\int_{100}^{200}\int_{300}^{350} 100x^{0.6}y^{0.4}dydx = 100\int_{100}^{200}\int_{300}^{350} x^{0.6}y^{0.4}dydx$$

We solve the first integral:

$$\frac{y^{1.4}}{1.4}x^{0.6}$$

We evaluate at the integration limits:

$$x^{0.6}\left[\frac{350^{1.4}}{1.4} - \frac{300^{1.4}}{1.4}\right] = x^{0.6}\cdot 505.38$$

We solve the second integral:

$$100\int_{100}^{200} x^{0.6}\cdot 505.38dx = 50538\int_{100}^{200} x^{0.6}dx$$

$$50538\frac{x^{1.6}}{1.6}$$

We evaluate at the integration limits:

$$50538\left[\frac{200^{1.6}}{1.6} - \frac{100^{1.6}}{1.6}\right] = 50538\cdot 2012.25 = 101695090.5$$

Now we divide this number by the area of the rectangle: $(200-100)(350-300) = 5000$.

$$\text{Average Production:}\frac{101695090.5}{5000} = 20339.18$$